

# Wine productivity per farm size: A maximum entropy application<sup>☆</sup>

Aníbal Galindro<sup>a,\*</sup>, Micael Santos<sup>a</sup>, Cátia Santos<sup>a</sup>, Ana Marta-Costa<sup>a</sup>, João Matias<sup>b</sup>,  
Adelaide Cerveira<sup>c</sup>

<sup>a</sup>University of Trás-os-Montes e Alto Douro (UTAD, [www.utad.pt](http://www.utad.pt)) and Centre for Transdisciplinary Development Studies (CETRAD), Quinta de Prados, 5000-801 Vila Real, Portugal

<sup>b</sup>University of Trás-os-Montes e Alto Douro (UTAD, [www.utad.pt](http://www.utad.pt)), and CMAT-UTAD, Portugal

<sup>c</sup>University of Trás-os-Montes e Alto Douro (UTAD, [www.utad.pt](http://www.utad.pt)), and INESC-TEC, Portugal

Received 28 September 2017; received in revised form 29 December 2017; accepted 1 March 2018

Available online 27 April 2018

## Abstract

The size of a farm is one of the factors that influence its productivity, in an ambiguous relationship that is often discussed in the industrial economy. In Portugal, the Demarcated Douro Region (DDR) is characterized by very small farms. Usually, this trend is considered a limiting factor in the profitability of the wine farms. In order to assess the correctness of this sentence, the variation of wine productivity per land size, from 2010 to 2016, was studied in the DDR, considering its three distinctive areas: Baixo Corgo, Cima Corgo and Douro Superior. The farms were categorized in nine different size ranges; as these variables outnumber the available seven observations, the Generalized Maximum Entropy (GME) estimator was used, since it suits the need to solve an ill-conditioned problem. GME was applied with the MATLAB (MATrix LABoratory) software along with the Bootstrap technique. According to the simulations, larger farms (with an area greater than 20 ha) on Douro Superior and Cima Corgo reveal higher marginal productivity given the current state of the region. On the other hand, Baixo Corgo's results suggest that medium-sized farms (with area ranges between 2 and 5 ha) display higher marginal increments to the region wine productivity. © 2018 UniCeSV, University of Florence. Production and hosting by Elsevier B.V. This is an open access article under the CC BY-NC-ND license (<http://creativecommons.org/licenses/by-nc-nd/4.0/>).

**Keywords:** Maximum-entropy; Wine productivity; Farm size

## 1. Introduction

Companies' size may influence their economic performance (Baumol, 1967) and it can also be a competitive advantage. In the agriculture sector, the farm size affects the performance or productivity of the farm, but this relationship is somewhat controversial (Townsend et al., 1998).

The vineyard activity has a strategic importance for the Portuguese and the European agriculture sector. According to 2016 data, Portugal is the 11th world wine producer, the 9th world exporter in value and the 5th largest European producer regarding production volume (OIV, 2017).

In Portugal, wine production has a great tradition, particularly in the Demarcated Douro Region (DDR), the first viticulture region to be delimited and regulated worldwide, in 1756. The DDR is located in the Northeast Portugal, in the Douro river basin, surrounded by mountains. Due to the heterogeneity of climatic, topographic and soil characteristics, this region produces superior quality wines, most of them with Protected Denomination of Origin (PDO), including the unique and worldwide famous fortified Port wine. The DDR is divided into three sub-regions: Baixo Corgo, Cima Corgo and Douro Superior (Magalhaes, 1998), with approximately 250,000 ha of total area, 45,000 ha of which are occupied by continuous vineyards. The harvest of 2016/17, constituted 22% of the Portuguese total wine production (1,336,612 hl), being the most representative wine region of the country (IVV, 2017a).

The portuguese viticulture sector, including the three sub-regions of the DDR, is embodied mostly by farms with less than 5 ha (see Fig. 1). In Portugal, around 63% of the farms

<sup>☆</sup>Fully documented templates are available in the elsarticle package on CTAN.

\*Corresponding author.

E-mail address: [anibalg@utad.pt](mailto:anibalg@utad.pt) (A. Galindro).

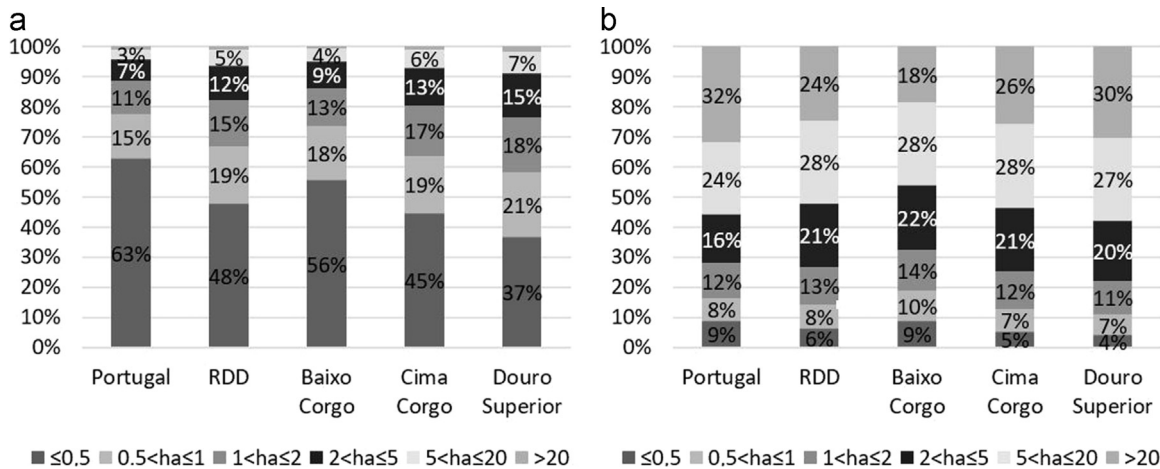


Fig. 1. Farm size in the viticulture sector of Portugal and DDR, in 2013, by classes of area. (a) Proportion of number of farms; (b) Proportion of farm area (Data from INE (2013) and IVDP (2010-2016a)).

have less than 0.5 ha, although they represent only 9% of the total area. Comparatively to the country, the DDR has fewer farms with less than 0.5 ha: 48% of farms, representing 6% of its total area. On the other hand, the farms with more than 20 ha constitute only 1% of the total number of farms, in Portugal and in the DDR, corresponding to 32% and 24% of their total area, respectively. Analysing the sub-regions of the DDR, the Baixo Corgo has the highest percentage of farms with an area between 0.5 and 5 ha, both in number and in total area, while the Douro Superior presents the lowest. Additionally, from 1989 to 2015, the total vineyard area of the DDR suffered a slight decrease (IVV, 2017b). This could be due to shrinking profit margins and to European Union (EU) regulations, which supported the farmers for uprooting their vineyards and imposed a limit on new plantations (Meloni and Swinnen, 2013).

This work aims to analyse the influence of farm size on wine production in the three sub-regions of the DDR (Baixo Corgo, Cima Corgo and Douro Superior), considering the land productivity. To attain this, we explore some Generalized Maximum Entropy (GME) estimators, using nine classes of size area for the referred sub-regions. The article is divided in four sections. The first section presents an overview of the DDR and a brief literature review regarding the influence of farm size on productivity. Section 2 introduces the available data and describes the chosen methodology, supported by relevant references and mathematical formulations. In the two subsequent Sections 3 and 4, the GME results are presented and analysed. Finally, Section 5 summarizes the main conclusions and makes suggestions for further improvement.

## 2. Theory and calculation

Sellers and Alampi-Sottini (2016) analysed the influence of farm size on the economic performance of Italian wineries, using profit, productivity and efficiency measures on a sample of 723 wineries for the year of 2013. The results showed that the size of the farm is positively correlated with all indicators of performance and the company may achieve the optimum

size and higher efficiency, with increasing returns to scale, when the unitary costs are minimized (Sellers and Alampi-Sottini, 2016).

Other authors argue that the positive relationship between size and productivity is explained by the increasing returns to scale, which means that when a farm increases its size (input), the production (output) increases proportionally more (Diewert and Fox, 2010; Sheng et al., 2015).

In the agricultural sector, economies of scale are more commonly associated with mechanization, especially when linked to high-performance (Gleyses, 2007). The technological progress and the access to improvements can also explain why big farms are more productive, since they often have more capital available than small farms to invest in new technologies, which allow them to reach higher productivity levels (Hooper et al., 2002).

Another theory is that small farms may have more difficulties to conquer new emergent opportunities in the international market: when larger volumes of goods are required and the market competition increases, their low production capacity restrains a possible adjustment to these challenges (Commission, 2005; Sheng et al., 2015).

The positive relationship between the size and the performance or productivity of a company is not always confirmed. As observed by Marcus (1969) and by Capon et al. (1990), only some industries display that type of results. In addition, an inverse correlation between farm size and productivity has been detected (Berry and Cline, 1979), mainly concerning developing countries (Ghose, 1979; Chand et al., 2011; Chen et al., 2011). However, Ghose (1979) argues that the advances in technology are the main factor for the vanishing of this inverse correlation, while Townsend et al. (1998) showed that this relationship was weak and inconsistent.

The authors considered that the main mechanism contributing to the positive farm size and productivity relationship is the advantage of larger farms in obtaining financial and other non-labour inputs. In the viticulture sector, farm size increase may be a consequence of the elimination of small-scale producers (Kroll, 1987) or an effect of the growth of some farm holdings

by planting new vineyards (Delord et al., 2015). Choi et al. (2016) studied the production efficiency of emerging vineyards in the 14 Northern U.S. States and concluded that, even though there is a positive relation between productivity and farm size, a negative relationship was also found, which was more marked for the youngest vineyards. This trend is explained by the accumulated experience of vineyards with longer histories, which improves their production efficiency.

However, the productivity does not depend only on farm size but also on other yield enhancing inputs, such as fertilizers, crop choices, seed selection and access to irrigation and technology (Chand et al., 2011). Due to the unfeasibility of including all these inputs, land productivity is an indicator often used in studies (Ghose, 1979; Hossain and Hussain, 1977), as a partial measure of productivity, because it relates the output to a single input. Coelli et al. (2005) and Townsend et al. (1998) include all inputs by using the total factor productivity.

The GME estimator (which will be explained with more detail on Section 2.1) has a spectrum of applicability that resembles the scope of more classical methods, such as the Ordinary Least Squares (OLS) and Maximum Likelihood Estimation. Macedo et al. (2014) compared the performance of GME estimators against those classical methods, using the well-established theory of state-contingent production. They found that the maximum entropy based estimators may be powerful tools, since the GME estimated mean of technical efficiency presented very small mean squared losses and small differences between the real data and the simulation. Moreno et al. (2014) applied the GME methodology with success, to quantify the impact of fuel costs on electricity prices for Spanish industrial consumers. Besides the GME, Fraser (2000) also used the Generalized Cross Entropy (GCE) estimator in a study on meat demand in the United Kingdom, in order to overcome the high collinearity among the explanatory variables. Sriboonchitta et al. (2015) used a latent variable model on a consumer willingness-to-pay case study; as this model is usually associated with several difficulties in the error distribution specification, the GME was used to solve that problem. Similarly to these last two studies, our reasons to use GME involve surpassing an inherent model problem: more variables than observations. Our study tries to highlight the usefulness of such a methodological approach in a problem with a limited dataset. Even though the DDR assumes itself as an important wine production region, the limited amount of available data hinders the application of models to provide useful recommendations. This study tries to validate GME as a suitable way to work with such small datasets.

### 2.1. Data and methodology

To achieve the purpose of this work, we used the available data from the Port and Douro Wines Institute (IVDP - Instituto dos Vinhos do Douro e Porto) (IVDP, 2010–2016a, 2010–2016b), between 2010 and 2016. The data categorizes the farm size within the three sub-regions of Douro (Baixo Corgo, Cima Corgo and Douro Superior) in nine different groups. Summarily, we have

seven observations (yearly data from 2010 until 2016) from each Douro's sub-region, 21 in total, and nine variables referring to farm sizes intervals, in hectare: [0,0.1]; ]0.1,0.5]; ]0.5,1]; ]1,2]; ]2,5]; ]5,8]; ]8,10]; ]10,20] and higher than 20.

From the OLS methodology point of view, our featured problem is underdetermined. This situation can be surpassed using the Maximum Entropy (ME) methodology, namely the GME estimator, widely used in problems where the number of unknown parameters exceeds the number of observations, and in models with small samples sizes. Some examples of applications of the ME principle can be found in Dionísio et al. (2008), Golan and Dose (2001), Miller and Horn (1998).

Taking the concept of entropy developed by Shannon (1948), Jaynes (1957a, 1957b) proposed the ME approach, which seeks information within the data without imposing arbitrary restrictions. Taking that in consideration, Golan et al. (1996) developed the GME and the Generalized Cross Entropy (GCE). Golan and Perloff (2002) suggested the Generalized Maximum Entropy- $(\alpha)$  (GME- $(\alpha)$ , where  $\alpha$  accounts for an higher order GME) to replace the (Shannon, 1948) formulation of entropy for Rényi (1970) and Tsallis (1988). Since those estimators managed to overcome the lack of data problem, they became a viable alternative for the classical approach OLS.

Excessive variables regarding the high polynomial extension may be avoided by other methods, such as the different types of Neural Networks (NN) (see Gurney, 1997 for further information), which have been extensively used in real-world applications. Nonetheless, since our main goal is to achieve a straightforward interpretation of the estimated coefficients, instead of producing an input-output forecast, we found GME to be more suitable than NN to formulate our problem. Due to the great adaptability of GME (and the other previously mentioned ME estimators), the years that followed his creation showed a legitimate spectrum of applications in the economics field, as stated on the previous section.

### 2.2. Generalized maximum entropy estimator

In this subsection we will briefly overview the ME and GME estimator features. According to the ME principle developed by Shannon (1948) and Jaynes (1957a, 1957b),  $X$  is a variable with possible outcome values  $x_k$ , with  $k$  going from 1 until  $K$ . Associated to those possible outcomes, there are  $p_k$  probabilities where  $\sum_{k=1}^K p_k = 1$ . We also define  $\mathbf{p}$  as a  $K$ -dimensional vector with the chosen probabilities  $p_k$ . Therefore, being  $y$  the average value of  $X$ ,  $y = E(X)$ , and given the ME principle, we can present the following objective function:

$$\max H(\mathbf{p}) = -\mathbf{p}^T \log \mathbf{p} \quad (1)$$

Subject to the following data consistency and additivity constrains:

$$y = -X^T \mathbf{p} \quad (2)$$

$$1 = -\mathbf{p}^T \mathbf{1} \quad (3)$$

where  $X$ ,  $\mathbf{p}$  and  $\mathbf{1}$  are  $K$ -dimensional vectors, being  $\mathbf{1}$  a vector of ones. The maximization problem can be solved analytically with

the Lagrangian Method (see Sokolnikoff and Redheffer, 1966). There is also a popular ME application called the “die problem”, which can be found extensively in the literature, although we refer Wu (2009) as a proper suggestion for its understanding. GME presents a generalized ME solution to the inverse problem of the regression framework. Regarding the following Eq. (4),  $y$  is a  $T$ -dimensional vector,  $X$  is a  $T \times K$  design matrix, both of them observed,  $\beta$  is a  $K$ -dimensional vector of unknowns, and finally  $e$  is a  $T$ -dimensional vector of disturbances, also unknown.

$$y = X\beta + e \tag{4}$$

It should be noticed that  $e$  compiles one or more sources of noise in the observed system. GME reparameterizes  $\beta_k$  as expectations of random variables with compact supports and  $M$  possible outcomes, where  $2 \leq M < \infty$ . Therefore,  $z_k = [z_{k1}, \dots, z_{kM}]^T$  is the mentioned support vector, with  $z_{k1}$  and  $z_{kM}$  representing the extreme values (lower and upper bounds).  $\beta_k$  can be expressed as a convex combination,  $\beta_k = \sum_{i=1}^M p_{ki} z_{ki}$ , with the positive coefficients  $p_k = [p_{k1}, \dots, p_{kM}]^T$  corresponding to the probabilities, whose values add up to one. The reparameterization is given by Eq. (5), where  $Z$  is an  $K \times KM$  matrix and  $p$  is an  $KM$ -dimensional vector of probabilities.

$$\beta = Zp = \begin{bmatrix} z_1^T & 0 & \dots & 0 \\ 0 & z_2^T & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & z_K^T \end{bmatrix} \begin{bmatrix} p_1 \\ p_2 \\ \vdots \\ p_K \end{bmatrix} \tag{5}$$

The reparameterization of the previously mentioned error term  $e$  is somehow analogous to the  $\beta$  representation in probability and compact supports,

$$e_t = v_t^T w_t, \tag{6}$$

where  $v_t = [v_{t1}, \dots, v_{tJ}]^T$  is a finite support for  $e_t$ , with  $J$  representing the possible outcomes  $2 \leq J < \infty$  of  $e_t$  as a finite and discrete variable, and  $w_t = [w_{t1}, \dots, w_{tJ}]^T$  is a  $J$ -dimensional vector of positive weights that add up to one (analogous to  $p_k$ ). The nuclear intention is to choose a set of error bounds,  $v_{t1}$  and  $v_{tJ}$ , such that, for each  $e_t$ , the value  $p = P(v_{t1} < e_t < v_{tJ})$  may be very close to one. Following the previous statement, we can also represent the matrix form, Eq. (7), for the error component ( $T$  unknown disturbances).  $V$  is a  $T \times TJ$  matrix and  $w$  is a strictly positive  $TJ$ -dimensional vector.

$$e = Vw = \begin{bmatrix} v_1^T & 0 & \dots & 0 \\ 0 & v_2^T & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & v_T^T \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_T \end{bmatrix} \tag{7}$$

Now, we can rewrite our initial model, Eq. (4), as:

$$y = X\beta + e = XZp + Vw \tag{8}$$

Then, the GME estimator is defined by the following optimization model with an objective function based on Shannon (1948) concept of Entropy and constrains.

$$\max H(p, w) = -p^T \log p - w^T \log w \tag{9}$$

Table 1  
Area variables considered in our simulation of model.

Variables	Farm Size (ha)
$\log(x_{1t})$	Area < 0,1
$\log(x_{2t})$	0.1 < Area ≤ 0.5
$\log(x_{3t})$	0.5 < Area ≤ 1
$\log(x_{4t})$	1 < Area ≤ 2
$\log(x_{5t})$	2 < Area ≤ 5
$\log(x_{6t})$	5 < Area ≤ 8
$\log(x_{7t})$	8 < Area ≤ 10
$\log(x_{8t})$	10 < Area ≤ 20
$\log(x_{9t})$	> 20 Area

subject to:

$$y = XZp + Vw \tag{10}$$

$$I_K = (I_K \otimes I_M^T) p \tag{11}$$

$$I_T = (I_T \otimes I_J^T) w \tag{12}$$

The set of restrictions comprises a model constraints (Eq. (10)) and two additivity constraints (Eqs. (11) and (12)), where  $\otimes$  denotes the Kronecker product. This optimization problem can be solved once more using the Lagrangian method (Sokolnikoff and Redheffer, 1966). After solving the optimization problem, and remembering that we intend to study the partial land productivity, we collect the coefficient values alongside each variable (specified on Table 2) that indicate how much each farm size (within 9 size intervals) contributed to the final production. Apart from the inherent questionable sample size, we attempt to measure partial land productivity assuming that the external factors remained unchanged during the time period considered. Therefore, each solution (coefficient value) measures how much a specific farm size range contributed to the total production. The single-handed interpretation of each coefficient may refer to “how much” does the productivity increases upon the percentage addition of more farms to the given range. Although narrowing the scope of this work, we are interested in collecting comparative information between farm sizes, instead of forcing numerically affirmative results in such limited data.

### 3. Results

In this section, computational results are presented and discussed. We applied the GME with the MATLAB software, separately for each Douro region, according to Macedo (2013). Each simulation compiles  $T = 7$  observations, which corresponds to the seven years, and  $K = 9$  independent variables, which correspond to the nine different farm size considered from INE (2013) (see Table 1).

The  $X$  matrix ( $7 \times 9$ ) of explanatory variables compiles on each entry the total area value for each correspondent size. The 7-dimensional vector of noisy observations  $Y$  assembles the wine production (litres). Our experiment overviews three different support intervals,  $z_1$ ,  $z_2$  and  $z_3$ , with  $z_k = [z_{k1}, \dots, z_{kM}]^T$  where  $M = 5$ , as suggested commonly on previous bibliography (Macedo et al., 2014; Moreno et al., 2014). The similarity between the

Table 2

Simulation results (coefficients) with support vector  $z_2 = [-1, -0.5, 0, 0.5, 1]^T$ , (\*)-Statistically significant results for a 5% level of significance.

Param.	Area (ha)	B. Corgo	C. Corgo	D. Superior
$\beta_1$	< 0.1 Area	0.041044	-0.049453	-0.072642*
$\beta_2$	0.1 < Area $\leq$ 0.5	0.24175*	0.17772*	0.21848*
$\beta_3$	0.5 < Area $\leq$ 1	0.27487*	0.23004*	0.28309*
$\beta_4$	1 < Area $\leq$ 2	0.30244*	0.27588*	0.32916*
$\beta_5$	2 < Area $\leq$ 5	0.3289*	0.30461*	0.3655*
$\beta_6$	5 < Area $\leq$ 8	0.30007*	0.30276*	0.34303*
$\beta_7$	8 < Area $\leq$ 10	0.2934*	0.26856*	0.29587*
$\beta_8$	10 < Area $\leq$ 20	0.3179*	0.30797*	0.36679*
$\beta_9$	> 20 Area	0.31886*	0.33004*	0.40286*

Table 3

Land productivity for each Douro's region, between 2010 and 2016: approximate wine production per hectare (in litres).

Region/Year	2010	2011	2012	2013	2014	2015	2016
Baixo Corgo	4236	3059	3739	3959	3569	4501	3428
Cima Corgo	3807	3386	3048	3625	3360	3712	3273
Douro Superior	2229	1720	1656	2296	2320	2410	2083

previously mentioned bibliography and our empirical work is extended to the error term reparameterization, since we also settle the number of points ( $J$ ) equal to three and we apply the 3-sigma rule to settle the  $v_i = [v_{i1}, \dots, v_{iJ}]^T$  interval, due to the lack of information regarding the error term.

To statistically validate our estimated coefficients, we applied the Bootstrap method (for further information see Efron and Tibshirani (1993) and Rizzo (2008)). The chosen support vector assembles three different values for each zonal simulation:

$z_1 = [-0.5, -0.25, 0, 0.25, 0.5]^T$ ,  $z_2 = [-1, -0.5, 0, 0.5, 1]^T$  and  $z_3 = [-2, -1, 0, 1, 2]^T$ . It should be emphasized that the support vector is the same for each variable. Both dependent (wine production in litres) and independent variables (total area from each pre-defined interval) are logarithmic. The error support vector is settled to  $v_i = [-1, 0, 1]^T$  according to the 3-sigma rule (Pukelsheim, 1994).

We have used three different  $z_k$  values but we only show the results for  $z_2 = [-1, -0.5, 0, 0.5, 1]^T$ , presented on Table 2, since the other results are quite similar.  $z_1$  and  $z_3$  simulation results are presented in Table A1 of Appendix A.

#### 4. Discussion

Regarding the simulation results, there is a similar behaviour among the three Douro regions. The coefficients present a nominal growing trend when the farms' size increase (from  $\beta_1$  to  $\beta_5$ ), corroborating increasing returns to scale as the size of the farm increases. However, from  $\beta_6$  to  $\beta_9$  the area contribution gets less volatile with a decreasing slope until  $\beta_7$  and increasing afterwards, which indicates a hybrid return to scale configuration for medium to large size farms. It is also worth to notice the negative coefficient on  $\beta_1$  (farms with less than 0.1 ha) for the Douro Superior region. A possible explanation for this is that small farms on Douro Superior reveal such light-productivity that actually harm the

overall wine production of the region. Both Cima Corgo and Douro Superior sub-regions present the highest coefficient for their larger farms ( $\beta_9$ , with more than 20 ha), as opposed to Baixo Corgo, for which the highest coefficient corresponds to the medium-sized farms ( $\beta_5$ , between 2 and 5 ha). The straightforward interpretation of these results indicates that both Cima Corgo and Douro Superior may benefit from larger farms to increase their overall marginal land productivity, whereas Baixo Corgo region current state seems to ask for more medium-sized farms.

It is important to refer that many variables and contingencies were excluded from this study, and that each region has a different status quo and productivity background (Table 3). In fact, the productivity can vary across regions, because of the resource quality (e.g. land quality), the climate and the enterprise mix (Productivity Commission, 2005). Despite the geographical proximity of the three sub-regions of the DDR, they are different in resource quality, orography and edaphoclimatic conditions, which explains the differences of productivity among them (IVDP, 2010–2016a, 2010–2016b). The previously mentioned mild inverse relationship may also be related to an important remark by Sen (1966), where he states that wage market distortions may benefit peasant or smaller farms. In fact, the labour productivity may be lower but it can actually favour land productivity scores (which is the scope of our study). Another possible explanation for the inverse productivity relationship can also be traced to problems of measurement, as stated by Desiere and Joliffe (2017): the inference of the inverse relationship findings may be attributed to the systematic overreporting of production by farmers on small plots, and the underreporting on larger plots. Nonetheless, since we cannot identify an incisive inverse relationship, we prefer to consider that result as an indication of the presence of possible unacquainted noise in the model formulation.

#### 5. Conclusion

This empirical work studied the variation of wine productivity per land size (within 9 different farm size ranges) in the DDR, considering its three distinctive areas (Baixo Corgo, Cima Corgo and Douro Superior), from 2010 until 2016. Given the limited data and the fact that we were facing an ill-posed problem, with more variables than observations, we chose the GME approach to estimate how different farm sizes affect the productivity on each one of the three Douro sub-regions. Even though the study is minimalistic, since there are many other features on the wine industry that determines the output, we present results that suggest increasing returns to scale roughly until two hectare farms; surpassing this farm size, the returns to scale show an hybrid back and forth innuendo.

According to the GME simulations, farms with an area greater than 20 ha, on Douro Superior and Cima Corgo, reveal higher marginal productivity given the current state of the region, differently to Baixo Corgo, where our results suggest that medium-sized farms (with an area between one and eight ha),

may provide higher marginal increments to the region wine productivity. The statement that farms with small areas display weaker marginal productivity might be correct and self-explainable, since those farms (with an area of less than two ha) have been disappearing or expanding.

Further studies should be performed in order to validate and optimize the farm-size structure of the Douro region, since many features and constraints were omitted on our GMES simulation. It is also reasonable to apply other ME estimators like GCE and an higher-order GME and compare the results obtained. The number of explanatory variables can be increased, and the problem would still be workable with GMES, but it would be also interesting to have it applied in bigger samples or in different locations.

Douro Superior sub-region, has had a limited wine-growing expression until now, but has been the subject of major investments, notably by the creation of new farms, some of which have a considerable size. This trend is due to the improvement of road accessibilities and to the very favourable orographic and climatic conditions for vine growing (Magalhães, 1998).

## Appendix A. Simulation results

Table A1

Simulation results considering the three different support vectors  $Z_1 = [-0.5, -0.25, 0, 0.25, 0.5]^T$ ,  $Z_2 = [-1, -0.5, 0, 0.5, 1]^T$  and  $Z_3 = [-2, -1, 0, 1, 2]^T$ .

	Douro Demarcated Region								
	Baixo Corgo			Cima Corgo			Douro Superior		
	$Z_1$	$Z_2$	$Z_3$	$Z_1$	$Z_2$	$Z_3$	$Z_1$	$Z_2$	$Z_3$
$\beta_1$	0.13819 (5.7842)	0.041044 (0.79117)	-0.025765 (-0.37531)	0.06145 (1.8274)	-0.049453 (-0.97215)	-0.11518 (-1.9907)	0.0060289 (0.18868)	-0.072642 (-2.7144)	-0.10789 (-3.814)
$\beta_2$	0.26754 (62.296)	0.24175 (19.711)	0.21659 (10.777)	0.21989 (26.524)	0.17772 (10.938)	0.14993 (6.2187)	0.26075 (39.883)	0.21848 (23.977)	0.21221 (13.602)
$\beta_3$	0.2806 (243.65)	0.27487 (97.93)	0.26849 (41.715)	0.24615 (88.776)	0.23004 (31.069)	0.21849 (11.067)	0.30087 (159.54)	0.28309 (66.743)	0.2797 (22.178)
$\beta_4$	0.29372 (155.02)	0.30244 (70.605)	0.31288 (26.733)	0.27036 (130.04)	0.27588 (66.211)	0.28427 (21.111)	0.3258 (178.77)	0.32916 (85.153)	0.34578 (25.212)
$\beta_5$	0.30852 (98.432)	0.3289 (42.049)	0.34418 (22.343)	0.28806 (85.394)	0.30461 (55.785)	0.31411 (37.282)	0.34673 (129.47)	0.3655 (106.42)	0.38128 (42.524)
$\beta_6$	0.28925 (69.589)	0.30007 (29.032)	0.29148 (13.634)	0.27773 (41.936)	0.30276 (25.252)	0.31963 (16.227)	0.33335 (97.312)	0.34303 (81.993)	0.34133 (33.167)
$\beta_7$	0.27498 (56.091)	0.2934 (24.748)	0.32713 (13.935)	0.257 (39.104)	0.26856 (21.204)	0.25451 (8.4187)	0.30687 (59.319)	0.29587 (27.371)	0.25054 (7.2035)
$\beta_8$	0.2986 (63.13)	0.3179 (21.165)	0.32985 (7.3316)	0.28329 (51.839)	0.30797 (28.244)	0.33111 (13.282)	0.34202 (73.515)	0.36679 (47.836)	0.39347 (17.683)
$\beta_9$	0.30308 (102.97)	0.31886 (38.231)	0.32987 (13.196)	0.2991 (55.219)	0.33004 (31.949)	0.35417 (18.55)	0.36588 (80.384)	0.40286 (65.714)	0.40691 (41.763)

## Acknowledgements

This work was supported by the R&D Project INNOVINE & WINE - Vineyard and Wine Innovation Platform - Operation NORTE -01-0145-FEDER-000038, co-funded by the European and Structural Investment Funds (FEDER) and by Norte 2020 (Programa Operacional Regional do Norte 2014/2020); and by the European Regional Development Fund (FEDER) through the Operational Programme for Competitiveness and Internationalisation - COMPETE 2020 Programme within project "POCI-01-0145-FEDER-006961", and by National Funds through the FCT as part of project UID/EEA/50014/2013. The authors acknowledge the suggestions provided by the participants at the "24th APDR Congress" (Covilhã, Portugal, 2017) in which a previous version of this paper has been discussed.

## Conflicts of Interest

The authors declare no conflict of interest.

## References

- Baumol, W., 1967. Macroeconomics of unbalanced growth: the anatomy of an urban crisis. *Am. Econ.* 57 (3), 415–426.
- Berry, R., Cline, W., 1979. Agrarian structure and productivity in developing countries. *Ann. Phys.* 24, 118–173. [http://dx.doi.org/10.1016/0003-4916\(63\)90068-X](http://dx.doi.org/10.1016/0003-4916(63)90068-X).
- Capon, N., Farley, J., Hoernig, S., 1990. Determinants of financial performance: a meta-analysis. *Manag. Sci.* 36 (10), 1143–1159. <http://dx.doi.org/10.1287/mnsc.36.10.1143>.
- Chand, R., Prasanna, P., Singh, A., 2011. Farm size and productivity: understanding the strengths of smallholders and improving their livelihoods. *Econ. Polit. Wkly.* 26 (26–27), 5–11 (doi:10.1.1.363.4414).
- Chen, Z., Huffman, W., Rozelle, S., 2011. Inverse relationship between productivity and farm size: The case of China. *Contemp. Econ. Policy* 29 (4), 580–592. <http://dx.doi.org/10.1111/j.1465-7287.2010.00236.x>.
- Choi, J., Lee, W., Gartner, W., 2016. Vineyards in northern U.S. states: Farm size and productivity relationship. *J. Distrib. Sci.* 7 (4), 53–61. <http://dx.doi.org/10.15722/jds.14.7.201607.53>.
- Coelli, T., Rao, D., O'Donnell, C., Battese, G., 2005. *An Introduction to Efficiency and Productivity Analysis*, 2nd Edition Springer Science & Business Media.
- Delord, B., Montaigne, E., Coelho, A., 2015. Vine planting rights, farm size and economic performance: Do economies of scale matter in the french viticulture sector?. *Wine Econ. Policy* 4 (1), 22–34. <http://dx.doi.org/10.1016/j.wep.2015.03.001>.
- Desiere, S., Joliffe, D., Land productivity and plot size: Is measurement error driving the inverse relationship?, Policy Research Working Paper 8134, 2017.
- Diewert, W., Fox, K., 2010. Malmquist and törnqvist productivity indexes: returns to scale and technical progress with imperfect competition. *J. Econ.* 101 (1), 73–95. <http://dx.doi.org/10.1007/s00712-010-0137-0>.
- Dionísio, A., Reis, A., Coelho, L., 2008. Utility function estimation: the entropy approach. *Physica A* 387 (15), 3862–3867. <http://dx.doi.org/10.1016/j.physa.2008.02.072>.
- Efron, B., Tibshirani, R., 1993. *An Introduction to the Bootstrap*, 1st Edition Chapman & Hall/CRC, New York.
- Fraser, I., 2000. An application of maximum entropy estimation: the demand for meat in the United Kingdom. *Appl. Econ.* 32 (1), 45–49.
- Ghose, A., 1979. Farm size and land productivity in indian agriculture – a reappraisal. *J. Dev. Stud.* 16 (1), 27–49 (doi:110.1080/00220387908421742).
- Gleyses, G., 2007. Rendement d'échelle et économies d'échelle en agriculture. In: T. editor (Ed.), Cemagref, Action 4, UMR G-Eau, Montpellier, p. 29, politiques publiques, aversion au risque des agriculteurs et demande en eau d'irrigation.
- Golan, A., Dose, V., 2001. A generalized information theoretical approach to tomographic reconstruction. *J. Phys. A* 34 (7), 1271–1283.
- Golan, A., Perloff, J., 2002. Comparison of maximum entropy and higher-order entropy estimators. *J. Econ.* 107 (1–2), 195–211. [http://dx.doi.org/10.1016/S0304-4076\(01\)00120-8](http://dx.doi.org/10.1016/S0304-4076(01)00120-8).
- Golan, A., Judge, G., Miller, D., 1996. *Maximum Entropy Econometrics: Robust Estimation with Limited Data*, 1st Edition John Wiley and Sons, New York.
- Gurney, K., 1997. *An Introduction to Neural Networks*, 1st Edition UCL Press, London.
- Hooper, S., Martin, P., Fisher, G.L., 2002. Farm size and productivity: where are the trends taking us?. *Aust. Commod.* 9 (3), 495–500.
- Hossain, M., Hussain, M., 1977. Farm size, tenancy and land productivity: an analysis of farm level data in bangladesh agriculture. *Bangladesh Dev. Stud.* 5 (3), 285–348.
- Instituto Nacional de Estatística (INE). Superfície das culturas permanentes (ha), Localização geográfica (nuts - 2013), Tipo (culturas permanentes) e Classes de área (cultura agrícola), ([https://www.ine.pt/xportal/xmain?Xpid=INE&xpgid=ine\\_indicadores&indOcorrCod=0003480&contexto=bd&selTab=tab2](https://www.ine.pt/xportal/xmain?Xpid=INE&xpgid=ine_indicadores&indOcorrCod=0003480&contexto=bd&selTab=tab2)), (accessed 30 May 2017).
- Instituto dos Vinhos do Douro e do Porto (IVDP). Produção de vinhos, 2010–2016b, (<http://www.ivdp.pt>), (accessed 17 March 2017).
- Instituto dos Vinhos do Douro e do Porto (IVDP). Caracterização das sub-regiões por dimensão das explorações, 2010–2016a, (<https://www.ivdp.pt>), (accessed 30 May 17).
- Instituto do Vinho e da Vinha (IVV). Evolução da Produção Nacional de Vinho por Região Vitivinícola, 2017b, (<http://www.ivv.min-agricultura.pt/np4/35/>), (accessed 27 February 2017).
- Instituto do Vinho e da Vinha (IVV). Evolução da área de Vinha -Portugal Continental, 2017a, (<http://www.ivv.min-agricultura.pt/np4/35/>), (accessed 27 February 2017).
- International Organisation of Vine and Wine (OIV). 2017 World Vitiviculture Situation, OIV Statistical Report on World Vitiviculture, 2017, (<http://www.oiv.int/public/medias/5479/oiv-en-bilan-2017.pdf>), (accessed 15 December 2017).
- Jaynes, E., 1957a. Information theory and statistical mechanics. *Phys. Rev.* 106 (4), 620–630.
- Jaynes, E., 1957b. Information theory and statistical mechanics. *Phys. Rev.* 108 (4), 171–190.
- Kroll, J., 1987. *Politique Agricole et Relations Internationales: les Enjeux en 380 France et Dans la CEE*, 1st Edition Syros, Paris, France.
- Macedo, P., Silva, E., Scotto, M., 2014. Technical efficiency with state contingent production frontiers using maximum entropy estimators. *J. Product. Anal.* 41 (1), 131–140. <http://dx.doi.org/10.1007/s11223-012-0314-y>.
- Macedo, P., 2013. Contributions to the theory of maximum entropy estimation for ill-posed models. (Ph.D. thesis), Universidade de Aveiro.
- Magalhães, N., 1998. *Enciclopédia dos Vinhos de Portugal*. 1st Edition, Edições Chaves Ferreira, Portugal.
- Marcus, M., 1969. Profitability and size of firm: some further evidence. *Rev. Econ. Stat.* 51 (1), 104–109. <http://dx.doi.org/10.2307/1926958>.
- Meloni, G., Swinnen, J., 2013. The political economy of european wine regulations. *J. Wine Econ.* 8 (3), 244–284. <http://dx.doi.org/10.1017/jwe.2013.33>.
- Miller, G., Horn, D., 1998. Probability density estimation using entropy maximization. *Neural Comput.* 10 (7), 1925–1938 (doi:10.1.1.49.5860).
- Moreno, B., Garça-Álvarez, M., Ramos, C., Fernández-Vásquez, E., 2014. A general maximum entropy econometric approach to model industrial electricity prices in Spain: a challenge for the competitiveness. *Appl. Energy* 135 (7), 815–824. <http://dx.doi.org/10.1016/j.apenergy.2014.04.060>.
- Productivity Commission, 2005. Trends in Australian agriculture, Productivity Research Paper. Commonwealth of Australia, Canberra.
- Pukelsheim, F., 1994. The three sigma rule. *Am. Stat.* 48 (2), 88–91. <http://dx.doi.org/10.1080/00031305.1994.10476030>.
- Rényi, A., 1970. *Probability Theory*, 1st Edition North-Holland Publishing Company, Amsterdam, Netherlands.
- Rizzo, M., 2008. *Statistical Computing with R*, 1st Edition Chapman & Hall/CRC, New York.
- Sellers, R., Alampí-Sottini, V., 2016. The influence of size on winery performance: evidence from italy. *Wine Econ. Policy* 5 (1), 33–41. <http://dx.doi.org/10.1016/j.wep.2016.03.001>.
- Sen, A.K., 1966. Peasants and dualism with or without surplus labor. *J. Political Econ.* 74 (5), 425–450.
- Shannon, C., 1948. A mathematical theory of communication. *Bell Syst. Technol. J.* 27, 379–423.

- Sheng, Y., Zhao, S., Nossal, K., Zhang, D., 2015. Productivity and farm size in Australian agriculture: reinvestigating the returns to scale. *Aust. J. Agric. Resour. Econ.* 59 (1), 16–38. <http://dx.doi.org/10.1111/1467-8489.12063>.
- Sokolnikoff, I., Redheffer, R., 1966. *Mathematics of Physics and Modern Engineering*, 2nd Edition McGraw-Hill, New York.
- Sriboonchitta, S., Liu, J., Sirisrisakulchai, J., 2015. Willingness-to-pay estimation using generalized maximum-entropy: a case study. *Int. J. Approx. Reason.* 60, 1–7. <http://dx.doi.org/10.1016/j.ijar.2015.02.003>.
- Townsend, R., Kirsten, J., Vink, N., 1998. Farm size, productivity and returns to scale in agriculture revisited: a case study of wine producers in South Africa. *Agric. Econ.* 19 (1–2), 175–180.
- Tsallis, C., 1988. Possible generalization of boltzmann-gibbs statistics. *J. Stat. Phys.* 52 (1–2), 479–487.
- Wu, X., 2009. A weighted generalized maximum entropy estimator with a data-driven weight. *Entropy* 11 (4), 917–930. <http://dx.doi.org/10.3390/e11040917>.