A Mathematical Model for Vineyard Replacement with Nonlinear Binary Control Optimization

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Abstract

Vineyard replacement is a common practice in every wine-growing farm since the grapevine production decays over time and requires a new vine to ensure the business sustainability. In this paper, we formulate a simple discrete model that captures the vineyard's main dynamics such as production values and grape quality. Then, by applying binary non-linear programming methods to find the vineyard replacement trigger, we seek the optimal solution concerning different governmental subsidies to the target producer.

1 Introduction

Laying in the south western part of Europe, a significant portion of Portugal's economy substantially relies on prolific tourism endeavours and solid agricultural systems. It is fair to say that the wine sector is one of the most important Portuguese agricultural sub-sectors, mainly due to its solid traditional background, as bulky statistics display on official records. In fact, the International Organisation of Vine and Wine (OIV) caps Portugal as the 11th greatest vineyard area and the ninth wine exporter (in value) worldwide OIV [1]. Even though these numbers don't seem too impressive, they represent a convincing effort taking into account that Portugal's territory is quite limited. Apart from the pragmatic worldwide rankings, the Portuguese vineyards deliver way more than a simple grape output since the oldest demarcated region was classified by UNESCO as a World Heritage in 2001. The country is well aware of this valuable heritage and the past few years revealed a set of meaningful efforts aiming the overall improvement of this sector. These attempts are ranged from direct investment (vineyard renewal, infrastructures) to academic oriented studies that intend to attain higher sustainability [2] or

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efficiency levels [3]. Aside from the more immutable orographic and edaphoclimatic factors, a vineyard holds a very complex set of variables that may be optimized in order to attain a more prolific output. The scope of our work is focused primarily on the vineyard replacement timing and inner multi-plot optimization. Therefore, the extensive set of components that characterize the entire vineyard system are omitted here.

There is no such thing as a static agricultural system: every crop needs fertilization, rotation or replacement. Vineyards are not an exception whatsoever. The core subject of this article falls deeply into a pure optimization effort: we intend to provide optimal multi-plot vineyard replacement in a given time-frame. For that a mathematical model is proposed. Several optimization tools to approach the problem were evaluated. Binary non-linear optimization methods were selected due to their suitability.

The remainder of this article is organized as follows. Section 2 gives account of previous work, while Section 3 presents problem's function fitting preceded by a brief description of the collected data. Our model and methodology is described in Section 4, followed by the presentation of the numerical results and discussion in Section 5. Finally, Section 6 gives the main conclusions of our article.

2 Literature Review

It is a fair assumption to state that farmers, traditionally, adapt their judgement in agricultural planning based on their previous experience [4]. However, the last century prolific innovation increased the agricultural productivity massively, making this sector a contributor to the overall economic development of several countries [5]. Even though the simplistic and traditional approaches from casual familiar farms are usually completely outclassed (productively) by the huge well-structured and highly technological explorations, there is still room to the small producers that are willing to enter the market and attain a marginal revenue from the activity [6]. As stated on previous section, available bibliography compile several and distinctive optimization attempts, generally adapted to the agricultural subset that the authors intend to study. Mixed-integer programming planning (MIP) can be found in models developed by authors such as Masini [7], with an application to the fruit industry. Troncoso and Garrido [8] applied MIP to forestry productions with logistic optimization, while Fonseca, Cerveira and Mota [9] developed a forest thinning and clear-cutting model considering a forthcoming five year planning horizon. Jena and Poggi [10] adopt an MIP model to study and identify an harvest planning in the Brazilian sugar cane industry. Regarding specifically the wine sector, Ferrer et al. [11] apply MIP to obtain an optimal schedule for wine grape harvesting. Because, frequently, the agricultural systems intend to maximize/minimize several variables simultaneously (e.g., a farmer may intend to maximize production while minimizing pollution or environmental impact), the multi-objective optimization approach is quite common [12]. Groot et al. [13] apply multi-objective optimization and prove its suitability in mixed farm design, accounting several production and environmental variables. Agricultural subsistence is also highlighted on the multi-objective optimization models of Klein et al. [14] and Banasik et al. [15], which respectively intend to adapt the farm system to climate changes and evaluate closed loops on supply chains.[a] The willingness to optimize a given system can also be attained with more straightforward approaches. Millar et al. [16] and García-Díaz et al. [17] use sampling methods, index formulation and comparative analysis to pheromone baited-traps and soil carbon optimization, respectively. Patakas, Noitsakis and Chouzouri [18] analyse the relationship between vine transpiration and water stress in order to optimize irrigation patterns. Atallah et al. [19] and Ricketts et al. [20] put their efforts into the optimal grapevine disease control in New York and California vineyards, respectively. The optimal control (OC) approach bundles multi-period optimization of the

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[a] Closed-loop supply chains are often defined with integrated management of processes that aim to reintroduce returned products, parts and materials into the supply chain.

dynamic programming with the cutting edge suitability to treat a large spectrum of problems [21]. The OC methodology became quite popular with applications in several areas, such as Economics [22, 23], Biology and Health related problems [24, 25]. Previous works also blend the OC with other methodologies, such as the Maximum-Entropy estimation [26] and Deep Learning approximation [27]. Since OC is quite adaptable, this theory has also been applied to agricultural endeavours, e.g., to overcome invasive species [28], to control pests in an optimal way [29], or to provide real-time greenhouse heating [30]. Logically, since grape growing is a subset of the agriculture endeavour, it is expected to see at least a few studies of OC dedicated, exclusively, to vineyard challenges. Surprisingly, to the best of our knowledge, Schamel and Schubert [31] presents the lonely attempt to bundle OC and vineyard/grapevine systems. They develop an OC model to attain the ideal crop thinning in viticulture, maximizing grape quality and quantity [31]. Moreover, they give a brief overview of optimization methods, noting the broad range of applications to the several subsets of the agricultural entrepreneurship. In contrast, our problem seeks a purely binary non-linear programming (BNLP) approach. Even though a few authors, such as Silva, Marins and Montevechi [32] or Arredondo-Ramírez et al. [33], actually applied BNLP to the sugar production and the design of water systems in agriculture, their problem formulation is still quite different from our own.

3 Collected data and function fitting

Before getting to the concrete model, this section describes briefly the available database and precedent grape quality/quantity function fitting. A face-to-face survey was conducted to compile extensive information related to almost every production input factor and coming output and revenues. Unfortunately, there are weighty difficulties gathering the data. Sometimes, the farmers are not available to provide an accurate response or, in the worst case scenario, they do not provide an answer at all. We were able to obtain 20 surveys exclusively from the Douro region. These surveys compile bulky farm information, from which we only extract four variables: vineyard with each inherent plot age and area, grape production (in tons), and revenues (in Euro). The age of each vineyard plot is irrefutably the key factor of the whole study, because we intend to investigate the optimal vineyard replacement age. To fulfil the analysis requirements, one needs to associate the grape quality and output per each vineyard age. In fact, the combined grape quality and quantity are directly correlated to the final revenue of the sampled farm. The main goal is to optimize the vineyard replacement timing, accounting that the owner intends to maximize their profit on a previously ranged (5, 10, 15 and 60 years) decision timeframe. To accomplish that purpose, we need to fit two functions that depend on the vineyard's age: Quality (Q) and Quantity (K). To achieve a trustworthy representation of the expected production (in Kg per hectare) for each vineyard age, we presume that this function should follow a concave down display. Higher forms of nonlinearity could be attempted to better fit the data, but it is well known that the grape production rises until a certain amount, which may also be steady for a while, and after some age it starts decaying. On the other hand, the quality of the grape is expected to steadily rise following a simple incremental function equation – see (2). Therefore, the polynomial degree was chosen to describe observable features of the vineyard and function fitting. According to the survey information, it is assumed that the vineyard productivity usually rises until a certain age, starting to decay at some point, when the vineyard gets older. It is also known that a vineyard does not start its production, from the commercial point of view, until it gets 5 years old. Therefore, to penalize our function, a few young vineyards with production value of zero were introduced. We fitted our data with a 2nd degree polynomial function using Matlab, obtaining a reasonable goodness of fit, precisely, a R-square of 0.8051 (see Appendix). Figure 1 shows the fitted function and the actual productivity data.

This work assumes, by acquainting the collected surveys, that the grape quality varies along the time
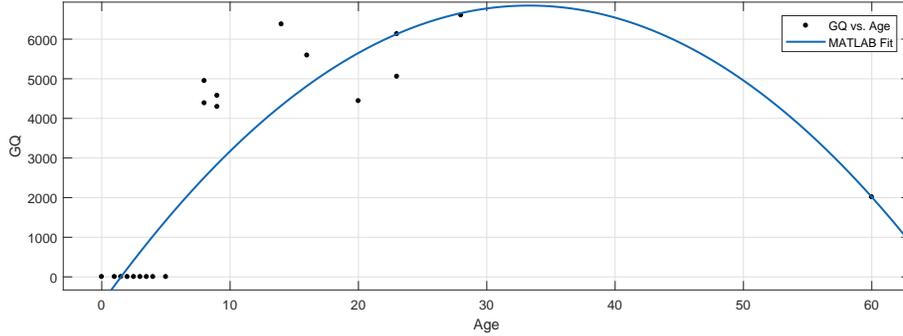
Fig. 1 Expected vineyard production per hectare (continuous–blue).

span. To get a hint about grape quality, the aforementioned data (production per hectare, Productivity) was recycled and linked with the total revenues from each farm. Equation (1) expresses the resulting grape quality variable (GQ):

$$GQ = \frac{TotalRevenues}{Productivity}. \tag{1}$$

Following the same premise of the grape quantity per age fitting function, it is assumed that the grape quality displays an expected pattern based on the collected surveys. This time we infer that the grape quality should linearly rise as the vineyard gets older. Performing an Ordinary Least Square regression in Python (see Appendix), to relate the quality proxy GQ with the vineyard age, we also applied a bootstrap re-sampling of 500 simulations to estimate our standard errors, confidence intervals, and to provide a graphical output: see Figure 2.

    The limited number of observations and possibly biased data (sometimes farmers do not provide accurate information) certainly harms our OLS (ordinary least squares) regression results (see Appendix). It is unavoidable to state that the obtained results in Appendix are not perfect, compiling not significant coefficients β_1 and a poor R-squared value. However, it is not our intention to perform a model that arbitrarily assumes values. Therefore, the calculated β_1 provides the least of two evils and is used in the next section. Quality Q and quantity K functions are computed according to (2) and (3), respectively:

$$Q(t) = 0.0036 I_t, \tag{2}$$

$$K(t) = -661.4 + 451.1 I_t - 6.774 I_t^2, \tag{3}$$

where I_t represents the vineyard age at time t. Since the vineyard produces once a year and a continuous decision time-frame is not realistic from producer's point of view, the problem formulation acquaints a discrete model. It is worth to note that the intercept of the regression is omitted in (2), because we are only interested in the evolution of the quality upon vineyard's age.

4 Methodology

In order to evaluate a real situation, we have selected a sample farm in the Douro region that relies its production on 5 different plots with the correspondent approximate area (in ha) and initial age per plot given by $A = [4.47, 1.45, 0.44, 1.66, 0.5]$ and $I_0 = [20, 30, 11, 5, 58]$, respectively. Let $T = \{0, \ldots, 59\}$ be the planning horizon with 60 one-year periods, where $t = 0$ corresponds to the first year and $P = \{1, \ldots, 5\}$ is the set of plots. Assigned to each plot $j \in P$, we consider the following parameters: Ii_j, the age of the stand in the first period; A_j, the area of the stand in ha.

    Considering equations (2) and (3), in our investigations the following constant values are considered:
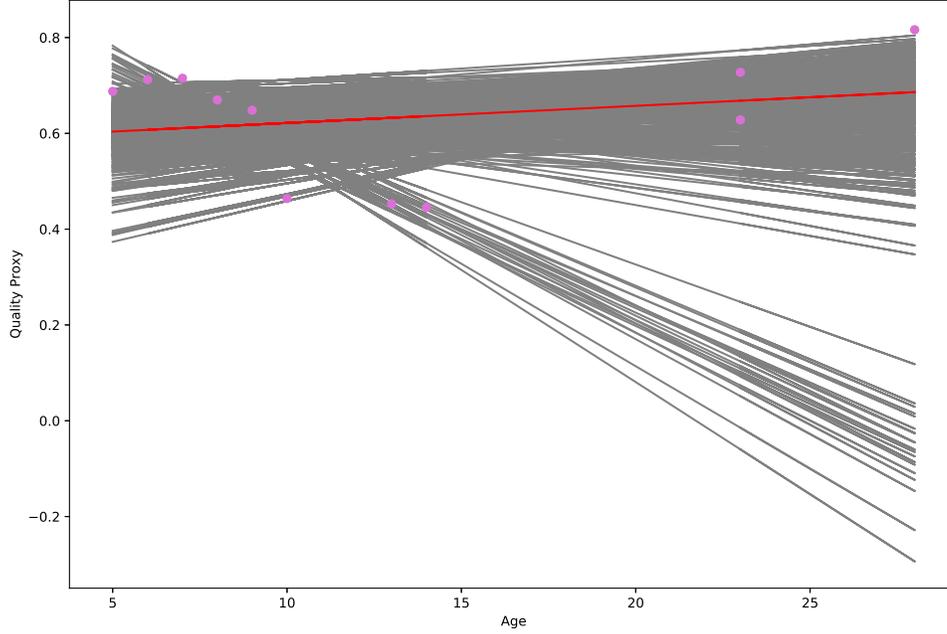
Fig. 2 Expected grape-quality improvement (continuous–red) obtained with 500 resampled bootstrap simulations (continuous–grey).

- $qc = 0.0036$, the quality constant;

- $p_0 = -661.4$;

- $p_1 = 451.1$;

- $p_2 = -6.774$;

- $Pu = 3$, the selling price of the grapes per Kg;

- $S = 10000$, cost of replacing the vine in Euro per hectare.

We also consider the following variables: u_j^t, binary variables taking value 1 if a cutting is performed in the plot $j \in P$ in the given period $t \in T$ and 0 otherwise; and Id_j^t, integer variables represent the age of the plot j in period t. We intend to maximize (4), which is our model's objective function,

$$\max \quad \sum_{t \in T} \sum_{j \in P} \left(Pu \cdot qc \cdot Id_j^t \cdot A_j \cdot (p_2 \cdot (Id_j^t)^2 + p_1 \cdot Id_j^t + p_0) - S \cdot A_j \cdot u_j^t \right), \tag{4}$$

subject to the constraints

$$Id_j^t = t + Ii_j - \sum_{\ell=0}^{t-1} u_j^\ell \cdot (Id_j^\ell + 1), \quad j \in P, \quad t \in T, \tag{5}$$

$$Id_j^t \in \mathbb{Z}_0^+, \quad j \in P, \quad t \in T, \tag{6}$$

$$u_j^t \in \{0, 1\}, \quad j \in P, \quad t \in T, \tag{7}$$

where (5) establish the age of each stand j at each period t and constraints (6) and (7) acquaint the variables domain. The objective function (4) compiles on the left positive size the variables that influence the farm's revenue, such as the grape price (Pu) and the age dependent grape quality (qc), alongside the production values given by the polynomial function $K(t)$ underpinned by the Area (A). The negative right-hand side discloses the expenses (augmented by the Area A) when the vine is cut. Therefore, the objective function (4) aims to maximize the farm profit in a given time-frame, optimizing the ideal timing of cutting (u).

Since the plots are not correlated with each other, the optimal replacement solution can be attained solving the general average profit function $L(t)$, given by (8), for a generic single plot:

$$L(t) = K_{Id(t)} \cdot Q_{Id(t)} \cdot A - s \cdot A, \quad t \in [0, +\infty[, \tag{8}$$

with

$$K_{Id(t)} = Pu \cdot qc \cdot Id_t, \quad Q_{Id(t)} = p_2 \cdot (Id_t)^2 + p_1 \cdot Id_t + p_0.$$

Function (8) heads up the farming revenues minus the fixed costs of renewing the vineyard:

$$Profit\ in\ the\ year\ 'n' \rightarrow L'(Id_n) = K_{Id(n)} \cdot Q_{Id(n)} \cdot A, \tag{9}$$

$$Profit\ in\ the\ cycle\ of\ 'N'\ size \rightarrow \hat{L}(N) = \frac{\sum_{i=0}^{N} L(i) - s \cdot A}{N}. \tag{10}$$

Proposition 1. *The solution is given by $N = 59$ when we look to the value of N that maximizes $\hat{L}(N)$. Replacing a generic vineyard at their 59 years of age, that is, $Id_t = 59$, gives the solution to the optimization problem in an infinite time horizon.*

Proof. Consider a single plot. Assume

$$P(Id_t) = Pu \cdot qc \cdot Id_t \cdot A \cdot \left(p_2 \cdot (Id_t)^2 + p_1 \cdot Id_t + p_0 \right)$$

and

$$c \cdot u_t = S \cdot A \cdot u_t.$$

Being $\sum_{i=0}^{59} u_i^* = 1$ the optimal replacement control for a single cut, we want to prove that for all I_t, $t \in [0, 59]$, does not exist $\sum_{i=0}^{59} u_i' \geq 2$ such that

$$P(Id_t^*) - c \sum_{i=0}^{59} u_t^* > P(Id_t) - c \sum_{i=0}^{59} u_t',$$

$$P(Id_t^*) > P(Id_t) - (b-1) \cdot c, \ \text{with} \ b \in \{2, 3, \dots, 60\},$$

where $L(Id_t^*)$ acquaints a full 59 year optimal cycle:

$$L(Id_t^*) + L(Id_S) > L(Id_t^*) + L(Id_S) - (b-1) \cdot c, \ \text{with} \ S \in \{0, 1, 2, \dots, 59\}, \quad V \in \{0, 1, 2, \dots, 59\},$$

$$L(Id_S) > L(Id_V) - (b-1) \cdot c.$$

Assuming that $L(Id_S) = \min\{L(Id_t)\}$ and $L(Id_V) = \max\{L(Id_t)\}$, which is the maximum revenue range among two farm status, we have

$$0 > L(Id_V) - L(Id_S) - (b-1) \cdot c.$$

Replacing with real values, and because the minimal b is $b = 2$, we get

$$0 > 2885.66 - (-2.34) - 10000,$$

$$0 > -7112.$$

The proof is complete.

5 Results and Discussion

The numerical simulations were performed in the FICO Optimization Xpress software with the appended code written in Mosel [34] (see Appendix). Due to the high non-linearity of the model, underpinned by the polynomial in the objective function and the variables Id_j^t and u_j^t, the software execution time exponentially rises with the time-frame. The maximum horizon $t \in [0, 59]$ optimal solution was performed iteratively for a single cut $\sum_{i=0}^{t} u_i = 1$ or none $\sum_{i=0}^{t} u_i = 0$ (61 combinations) and validated with Proposition 1. The shorter time-frame optimizations by the farmer were performed directly with the FICO Optimization Xpress software with stepwise simulations until the final time $t = 59$, changing the model initial conditions at each step (see Appendix). The producers total yield and plot replacement age, per each considered time-frame, are stated on Table 1. As expected, when the producer is not able

Table 1 Vineyard replacement age per plot, for each decision time-frame, and the resulting total yield in Euro.

Age	Plot 1	Plot 2	Plot 3	Plot 4	Plot 5	Total Yield
5 Years	70	70	None	None	73	691238,21
10 Years	71	71	None	None	69	686398,61
15 Years	59	60	66	None	64	782085,19
60 Years	61	44	None	None	58	793114,13
Infinite	59	59	59	59	59	755712,99

to foresee, in a sufficiently large time-frame decision, the overall yield is significantly smaller than in a broad optimization period. The difference between optimizing in a 15 years time-frame and 5 or 10 years is quite obvious since the larger period outperforms the shorter ones in roughly 100 000 Euro over the 60 year analysis. The optimal global solution (60 years) caps only a marginally better result than the stepwise 15 year analysis, while the infinite horizon solution (IHS) acquaints a sensibly average result. The IHS operates naively, cutting the vineyard every time it reaches the age of 59 years old, maximizing the average profit function. That routine does not consider the ending of the simulation, performing unessential vineyard replacements. Nonetheless, if the producer does not have a sufficiently large planning time-frame, it is better to rely his decisions on the IHS than short term optimizations. Another interesting result, is that the shorter time-frame of 5 years planning, slightly outperforms the 10 years solution. This result might be explained by the circumstantial initial conditions of the model.

Even though we find this problem interesting from the producers point of view, we want to evaluate this problem considering the governmental available policies. It is well known that the Portuguese government performs regular support to the Douro grape producers, usually providing a beneficial price and allowing the producer to sell their grapes at higher prices. There has also been recent policies intended to provide support for vineyard replacement efforts.

We now assume that the government fully supports every vineyard replacement from the producer. Sub-sequentially, (10) is solved without the associated fixed costs sA. We gather a new optimal infinite horizon solution with the ideal replacement age at 49 years old, which yields a better average profit due to the absence of fixed and improved replacement endeavors. The results of Table 2 display the trivial producer improvement (A) due to the governmental aid upon the helpless (B), considering the IHS average yield (Avg Yield), average replacement costs (Avg RC), average production (Avg Production) and, finally, the average governmental support. The goal is to find the beneficial grape selling price ($Pu + a$, with a being the benefit) that the government needs to provide in order to make the producer (B) match the average yield of producer (A). We note that the producers (A) and (B) are the same, since they both represent the sampled farm from Section 4. The only difference among them is the optimization criteria. Regarding the producer (B) production, which remains the same, the government

Table 2 Average values comparison (in Euro and Kg, Production) when the government provides, or not, free vineyard replacement.

IHS type	Avg Yield	Avg RC	Avg Production	Avg Gov Support
(A) IHS - NRC	13633	1738	42316	1738
(B) IHS - RC	13027	1444	40985	None

needs to provide an additional 0.1257 cents per Kg to make this producer match the other one.

Table 3 Matched average yield from the producer with two different governmental policies.

IHS type	Avg Yield	Avg RC	Avg Production	Avg Gov Support
(A) IHS - NRC	13633	1738	42316	1738
(B') IHS - RC	13633	1468	41343	5167
(B) IHS - RC	13633	1444	40985	5151

As stated on Table 3, the governmental costs of this policy are way higher (almost triple) than the vineyard replacement support. Assuming that the producer previously knows about the governmental benefit, his IHS (10) shifts from the previous optimal to replace the vineyard a year earlier (58 years, labeled Producer (B')), but his option increases the production value making the governmental support per grape (Kg) decrease in order to match both producers average yield. Sub-sequentially, the smaller benefit makes the producer (B') shift again to the original optimal solution at 59 years replacement age. Nonetheless, we present both solutions on Table 3, even though they don't differ too much.

The developed model is purely deterministic: no forms of uncertainty were considered. Nonetheless, adding a stochastic representation to the model might deliver a more realistic approach if random phenomena, like the weather (temperature and rainfall) and disease proneness, were featured in the objective function (4).

Considering our problem formulation, and assuming that the farmers revenues depend directly on the produced grape quantity and quality, it is suggested that the government should provide direct and unequivocal vineyard replacement support instead of grape selling benefit. Assuming that the producer operates rationally and relies on a IHS, the replacement support induces a more prolific optimization, while the selling grape benefits are very unlikely to force optimal and incremental decisions from the producers, added to the fact that the government expenditure severely bloats with the latter tool.

6 Conclusions

We considered vineyard replacement activity as a crucial factor within the grape-growers optimization portfolio. The inner farm gross profit function was considered, strictly dependent from the produced grape quantity and quality functions, per each vineyard age alongside the fixed cost of replacing the vineyard. To obtain those functions, and simulate numerically a concrete illustrative example, 20 surveys were collected in the Portuguese Douro wine region. It was concluded that the producers optimization horizon plays an important role in the overall business profitability, since shorter forecasts usually lead to smaller income. An infinite horizon solution (IHS) was also calculated based on the average profit function (10), even though this solution, due to its naiveness (the age to replacement is fixed), depreciates when the optimization period is shorter. It was found that the IHS still outperforms short decision planning (5 and 10 years) in an overall 60 year time-frame. The important remark is that if the producer is not able to acquaint a sufficiently large period (at least 15 years) when optimizing

his vineyard replacement endeavours, it is preferable to rely purely on the IHS.

Afterwards, governmental intervention was considered on producers that follow the IHS. Namely, we investigated two possible supporting tools: providing a grape selling price benefit or free vineyard replacement to the producer. The results were compared and evaluated. The second tool is the prominent method, because it induces the producer to seek freely the vineyard replacement age, maximizing his overall production without considering the fixed costs. To match the producers that receive a free replacement scenario (average profit), the government benefit to the grape selling price reveals itself way more expensive and roughly innocuous upon the producer optimization decision.

Our model and study, even though based on real data, assumes and also omits many parameters that could be considered when developing a vineyard replacement strategy, which may also vary within different producers. For further developments, the database could be extended alongside the input variables on the objective function, to explain the problem more accurately and also to increase the spectrum of governmental policies (e.g., beneficial labour or provide mechanization). There is also room to attempt new robust optimization methods such as RCMARS, generally applied to optimize trade-off between risk and return in the financial market [35]. An analogous problem formulation may allow the junction of data-mining methods. On the other hand, there are other chances to model the problem with an increasing level of complexity. The independence assumed in our variables make the nonlinear binary control viable. However, if somehow it becomes valuable to entangle the quality and quantity variables and introduce stochastic effects, such us the weather or vineyard diseases, approaches such as bang-bang control optimization [36] and stochastic optimal control under a switching regime [37,38] might be useful tools for further endeavours.

Acknowledgements

Appendix

We have used Matlab R2017a to fit our data with a 2nd degree polynomial, with Figure 1 showing the fitted function and the real productivity data. Follows our Matlab code:

```
function [fitresult, gof] = createFit1(Age, GQ)
%CREATEFIT1(AGE,GQ)
%  Create a fit.
%
%  Data for 'MATLAB Fit' fit:
%      X Input : Age
%      Y Output: GQ
%  Output:
%      fitresult : a fit object representing the fit.
%      gof : structure with goodness-of fit info.
%
```

```
[xData, yData] = prepareCurveData( Age, GQ );

% Set up fittype and options.
ft = fittype( 'poly2' );
opts = fitoptions( 'Method', 'LinearLeastSquares' );
opts.Robust = 'LAR';

% Fit model to data.
[fitresult, gof] = fit( xData, yData, ft, opts );

% Plot fit with data.
figure( 'Name', 'MATLAB Fit' );
h = plot( fitresult, xData, yData );
legend( h, 'GQ vs. Age', 'MATLAB Fit', 'Location', 'NorthEast' );
% Label axes
xlabel Age
ylabel GQ
grid on

Results of Fitting:

Linear model Poly2:
f(x) = p1*x^2 + p2*x + p3
Coefficients (with 95% confidence bounds):
p1 =      -6.774  (-8.864, -4.685)
p2 =       451.1  (332.2, 570)
p3 =      -661.4  (-1674, 351.1)

Goodness of fit:
SSE: 2.678e+07
R-square: 0.8051
Adjusted R-square: 0.7822
RMSE: 1255
```

To relate the quality proxy GQ (1) with the vineyard age, and to do a bootstrap re-sampling of 500 simulations to estimate standard errors, confidence intervals, and to obtain Figure 2, we have used Python 3.5.3 with the following code:

```
import pandas as pd
import numpy as np
from cairocffi import *
import matplotlib
matplotlib.use('Gtk3Agg')
import matplotlib.pyplot as plt
df = pd.read_excel('___.xlsx')   #UPLOAD THE DATABASE FILE
age = df['age']
quality = df['quality']
from sklearn.linear_model import LinearRegression
X = np.vstack([age, np.ones(len(age))]).T
```

```
plt.figure(figsize=(12,8))
for i in range(0,500):
sample_index = np.random.choice(range(0, len(Bundle)), len(Bundle))
X_Samples = X[sample_index]
Y_Samples = Bundle[sample_index]
lr = LinearRegression()
lr.fit(X_Samples,Y_Samples)
plt.plot(Idade, lr.predict(X), color='grey', alpha=0.2, zorder=1)
plt.scatter(Idade,Bundle, marker='o', color='orchid', zorder=4 )

lr = LinearRegression()
lr.fit(X, Bundle)
LinearRegression(copy_X=True, fit_intercept=True, n_jobs=1, normalize=False)
plt.plot(Idade, lr.predict(X), color='red', zorder=5)
plt.xlabel('Age')
plt.ylabel('Quality Proxy')
plt.savefig("/home/pi/Desktop/Anibal_Docs/scatter3.png", dpi=125)

import statsmodels.formula.api as smf
results = smf.ols('Bundle ~ Idade',data=df).fit()
print(results.summary())

OLS Regression Results
==============================================================================
Dep. Variable:                 Bundle   R-squared:                       0.051
Model:                            OLS   Adj. R-squared:                 -0.054
Method:                 Least Squares   F-statistic:                    0.4849
Date:                Thu, 12 Apr 2018   Prob (F-statistic):              0.504
Time:                        09:51:11   Log-Likelihood:                 8.0719
No. Observations:                  11   AIC:                            -12.14
Df Residuals:                       9   BIC:                            -11.35
Df Model:                           1
Covariance Type:            nonrobust
==============================================================================
coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
Intercept      0.5858      0.078      7.468      0.000       0.408       0.763
Idade          0.0036      0.005      0.696      0.504      -0.008       0.015
==============================================================================
Omnibus:                        2.482   Durbin-Watson:                   2.092
Prob(Omnibus):                  0.289   Jarque-Bera (JB):                1.450
Skew:                          -0.651   Prob(JB):                        0.484
Kurtosis:                       1.788   Cond. No.                         31.0
==============================================================================
```

The numerical simulations reported in Section "Results and Discussion" were performed in the **Xpress-Mosel** multi-solver modeling and problem solving environment. Follows our code for the first simulation of the producer with a time-frame of 5 years (the code for the other simulations is similar):

```
model OptimizationVine
```

```
uses "mmxnlp"

declarations
nT=4  !n year planning
T=0..nT !planing horizon
nd=5         !total number of plots
P=1..nd  !plot set
qc=0.0036   !quality constant
p0=-661.4   !quantity constant
p1=451.1
p2=-6.774
Pu=3         !grape selling price (kg)
S=10000      !vineyard replacement cost (per hectare)

Ii: array(P) of integer
A: array(P) of real

u: array(P,T) of mpvar
Id: array(P,T) of mpvar

end-declarations

Ii::[20,30,11,5,58] !Each plot age at 2017 (or T=0)
A::[4.47,1.45,0.44,1.6,0.5] !Each plot area

forall(j in P,t in T) do
create(u(j,t))
u(j,t) is_binary
end-do

forall(j in P,t in T) do
create(Id(j,t))
Id(j,t) is_integer
end-do

!ORIGINAL
objective:= sum(t in T,j in P)
(qc*Id(j,t)*A(j)*Pu*(p2*(Id(j,t))^2+p1*Id(j,t)+p0)-S*A(j)*u(j,t))

!constraints

forall(j in P) do
sum(t in T) u(j,t) <= 1
end-do

forall(j in P,t in T) do
Id(j,t)=t+Ii(j)-sum(l in 0..t-1) (u(j,l)*(Id(j,l)+1))
end-do
```

```
setparam("XPRS_verbose", true);
maximize(objective);

forall(j in P,t in T) do
if(getsol(u(j,t))<>0)
then writeln("Plot ",j," is replaced at period ",t," u(",j,",",t,")=",getsol(u(j,t)))
end-if  !writes only if a plot is replaced
end-do


forall(j in P,t in T) do
writeln("Plot age ",j," at period ",t," ID(",j,",",t,")=",getsol(Id(j,t)))
end-do                 !retrieves the age of each plot per period


forall(t in T) do
writeln("Yield of period ",t," =", sum(j in P)
(qc*getsol(Id(j,t))*A(j)*Pu*(p2*(getsol(Id(j,t)))^2
+p1*getsol(Id(j,t))+p0)-S*A(j)*getsol(u(j,t))))
end-do      !farm yield per period

writeln("Total Profit = ", getobjval, "\n")  !overall profit

end-model
```

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