THE USE OF *FUNCTON MACHINE:* AN EXPERIENCE IN THE CLASSROOM

Ana Paula Gandra[1], Ana Paula Aires[2], Paula Catarino[2]

[1] Escola Secundária Fontes Pereira de Melo, ESFPM (PORTUGAL)
[2] Departamento de Matemática da Universidade de Trás-os-Montes e Alto Douro, UTAD
Lab_DCT do CIDTFF da Universidade de Aveiro (PORTUGAL)

Abstract

The transition from Arithmetic to Algebra is one of the great difficulties of the students and there are errors that occur and persist over time. The curricular guidelines set out in the Principles and Standards for School Mathematics [1] suggest that students should significantly learn Mathematics based on their math skills that were previously acquired.

In Portugal, the Mathematics Program of Basic Education [2] considers that, in the 7th grade of basic school, students begin with formal study of algebra and when the students work with "letters" and points out, those students should understand the meaning of "letters", usually called variables and operations with them.

The chapter "Functions" awaken the algebraic thinking and leads, for example, to the generalization. In the case in the teaching process, there is a consensus that the issue of "Functions" must has a central place in algebra education, particularly at primary (elementary) level [3].

This paper explores the use of an applet, named *Function Machine*, which allows students to observe patterns and regularities from pairs of numbers and make generalizations using words and symbolic rules. As a specific purpose it was intended to show how an educational experience based on the observation of numerical regularities, allows an expression of general and contributes to the development of algebraic thinking.

Methodologically, we opted for a qualitative and interpretative research approach, based on a case study. Data collection was done through direct observation of a lesson and written productions of students. An activity was drafted with two issues and applied to two classes of the 7th grade of basic school during a mathematical class of fifty minutes, who answered individually to the data items and also interacted with the computer at the same time. The teacher is an author and assumed the role of observer.

The results show that most of these students from the 7th grade of basic school are enabled to make generalizations, find an expression that translates the general structure of the displayed pattern and justify their answers using the current language, the language of mathematics, or a combination of both languages. Sometimes the *Function Machine* influences the choice of the general expression of a pattern, because the student can confirm whether it is correct or not by viewing this dynamic form of information on the computer screen.

Overall, it is concluded that there was a significant improvement in student performance at the level of generalization.

Keywords: Pattern. Function. *Function Machine*. Algebraic thinking.

1 INTRODUCTION

As well as teachers, the educators in general and those responsible for education in each country believe is important the teaching of algebra, which explains why this content is part of all curricula in all countries. In Portugal, students begin the study of algebra in the 7th grade of basic school, under the topic of “Functions” with the introduction of the concept of variable [2]. The study of functions follows the work already done on the theme of sequences. It is possible identify regularities, analysing what happens to the values of the dependent variable at the same time as the independent variable suffers a given increase.

There are different ways to introduce students to the algebraic thinking. The study of generalization patterns has been pointed out by several studies (eg. [4], [5], [6], [7], [8], [9], [10]) where the authors

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believe that it is important for students create algebraic expressions or mechanisms leading to these, giving meaning to the use of symbols and algebraic thinking.

The first approach of algebra can be crucial to simplify or hamper the integration of the student in Algebra study [11]. Educators have the conviction that the ideas that students build about this matter are important to the success of this learning content. In Portugal, the big problem of the teaching of mathematics, "is that it does not promote, as required, the ability to think mathematically and using mathematical ideas in many contexts" [12, p.24]. According to what has been seen in the results of intermediate tests and national exams, algebra points out as one of the thematic areas where there is greater need to increase the teaching intervention.

According to the NCTM, there are several advantages of using technology in mathematical classes of basic education and refer that "technology is essential in teaching and learning of mathematics" and "influences the mathematics that is taught and enhances students' learning" allowing them to concentrate" on decisions to make, reflection, reasoning and problem solving "[13, p. 26].

In the 7th grade of basic school, a function must be seen as a rule. With the purpose of making the idea available to students, we opted for the use of an *applet* available on the Internet named *Function Machine*, as a tool for developing understanding of the concept of function. The *applets* are interactive virtual animations focused on the development or consolidation of a particular topic. The student puts values on the machine, objects, and sees the values that come out of it, images, recognizing the underlying rule.

In order to analyse the development of algebraic thinking of the 7th grade of basic school students, in the teaching unit "Functions", we developed a study on the ability to generalize. In this study, the students used words and symbolic rules to describe patterns and regularities from numbers and it was found that the use of the *applet*, *Function Machine*, allowed a faster and autonomous work by students.

This paper is organized as follows: the next section is devoted to a brief theoretical context of support to our study, followed by a section where we discuss the methodology adopted. In the fourth section we present the results obtained with this study and we concluded this article with a section where we expose some pertinent conclusions and we made some reflections about the obtained results.

2 THEORETICAL BACKGROUD

At the primary level, students must learn algebra "as a set of concepts and skills associated with the representation of quantitative relations and also as a mathematical reasoning style used to formulate standards, functions and generalizations "([13], p. 263). The Mathematics Program of Basic Education [2] refers the primary purpose of teaching algebra in the basic school is to develop in students the language and algebraic thinking.

Several authors (eg. [14], [15], [16], [17], [18], [19]), who worried about the learning difficulties of the students raised the level of algebra, have developed several studies to find a more productive way of working this topic. Currently, either the NCTM ([13]) or the Mathematics Program of Basic Education ([2]), give great prominence to the study of patterns, assumed their potential in the variable notion of learning.

Research on the development of algebraic thinking shows the importance of numerical investigations focused in the search for rules, in exploiting regularities and patterns, in the generalization and the use of multiple representations (eg. [20], [21]).

Kieran ([22]) refers the importance of generalization, considering the algebra as a way of thought, and points out that in very young children, the generalization arises from bridges established between arithmetic and algebra. This author considers it crucial, in generalization process, observing the sequential structure of operations, generated from the study of particular cases that show the way, paving the way for generalization.

Kaput ([5]) states that Algebra is based on four components: the study of functional relationships; the study and generalization of patterns and numerical relationships; the development and manipulation of symbolism; and modelling. Therefore, and in order to create favourable conditions for the full learning of Algebra, the NCTM ([13], p.39) names five standards for Algebra around which to organize the work to be done with students of all education levels:

a) Understand patterns, relations and functions;

b) To represent and analyse mathematical situations and structures using algebraic symbols;

c) Use mathematical models to represent and understand quantitative relationships;

d) To analyse the variation in different contexts.

According to this line of thinking also the Currículo Nacional do Ensino Básico ([23]) considered that the mathematical competence over the cycles is based on the predisposition that students must have to explore situations that they are problematic to look for regularities, to explore numerical and geometric patterns, to formulate and test conjectures, to formulate generalizations, to represent and interpret information in different ways and to think logically. In particular, in the field of Algebra on the basic school, there are specific issues to be addressed, namely "the understanding of the concept of function and facet which may represent, as a correspondence between sets and relationship between variables" ([23], p. 67).

In order to improve the development of algebraic thinking, it is important to develop a sense symbol ([24]). A necessary condition for this to happen is the use of appropriate teaching practices where all the work is developed through investigative and exploratory tasks, where students have the opportunity to explore patterns and numerical relationships and the ability to explain their ideas and where they can discuss and reflect on them. Patterns help the students to realize the "true" variable notion, which for most is just seen as an unknown number ([25]).

Blanton and Kaput ([26]) consider that the development of functional thinking, based on the analysis of patterns and relationships, opens ways to Algebra. It is a type of representational thinking that focuses on the relation between two varying quantities ([27]).

Kaput ([5]) believes that when students do not build their own knowledge, or when it is not given time to reflect on the learning, various errors and difficulties arise. On algebra learning, the main difficulties presented have to do with the fact that the symbols lose their connection to the concrete, which then is reflected in frequent errors in algebraic manipulation.

The way children think about the functional relationship has implications for subsequent learning, so it is necessary to perform tasks that allow deepen, extend and support the development of functional thinking of students, from the early years of education ([26]).

The use of technology is important and fundamental to the teaching of Mathematics as it enhances student learning ([1], [23]). There are specific programs to work this or that topic or concept, of which the best known examples are the *applets*, many of which are available on the Internet. These programs, which sometimes take the form of games, are often very helpful in promoting the learning of specific aspects of Algebra ([28]).

The Mathematics Program of Basic Education ([2]) encourages the use of technology, but recommends that they should be used in a responsible way in order to enrich and improve opportunities for mathematics student learning. Students are taken to work at higher levels of generalization or abstraction, since with the use of technology the student can examine more examples or view different forms of representation, which manually would be lengthy, leading him to formulate conjectures ([13]).

On the potential of applets in the teaching and learning of mathematics, Figueiredo and Palha ([29]) consider that the *applets* have an interactive character and create a complicity environment where students feel comfortable to take risks, experiment and explore, it is the computer that validates their answers and not the teacher. The authors mention also "The ease of understanding of these *applets* that mainly appeal to the informal knowledge of students makes it possible to treat the concepts in a natural and intuitive way, making thus a solid foundation for a work, later, more formal." ([29], p.7).

Drijvers et al. ([30]), based on an investigation on the use of technology in the study of the concept of Function, refer that *applets* are a mean which promotes the establishment of relations between the various representations of a Function. However, there must be a strong connection between the *applet* and the paper and pencil tasks, to avoid the development of individual and disintegrated knowledge ([31]), and in order to assure a contextualized conceptual knowledge ([30]).

3 METHODOLOGY

This exploratory study of qualitative and interpretative nature, intended to try to understand to what extent the use of an *applet, Function Machine*, that allows the observation of numerical regularities, in the teaching of the issue of the “Function”, can help students develop algebraic thinking and also help them when they attempt to make sense of an algebraic expression, or a letter in this expression, or when they try to write symbolically a certain generalization.

The participants of this study were forty-six students (twenty-four males and twenty-two females) of two classes of 7th grade of basic school, taught by one of the researchers (first author). The average student's age is 12 years and the ages were between 11 and 14 years old.

This study was made in a mathematical class with the duration of fifty minutes, in December 2015. The work was accompanied by direct observation and all written registers of students were collected.

The students' work were focused on solving an activity with two questions that concerned the search for numerical patterns and regularities and the formulation of generalizations, using the *Function Machine*, available in www.shodor.org/interactivate/activities/. The choice of this *applet* owed to the fact that it allows the connection between a regular numeric and symbolic writing the respective generalization algebraic expression, from the observation of pairs of numbers (Figure 1). The *Function Machine* allows students to explore situations with a large number of examples, since they can assign any values for the independent variable X, and obtains a value for the dependent variable Y. When students do the generalization writing an algebraic expression, if they wish, they can get feedback reply (correct answer or try again).

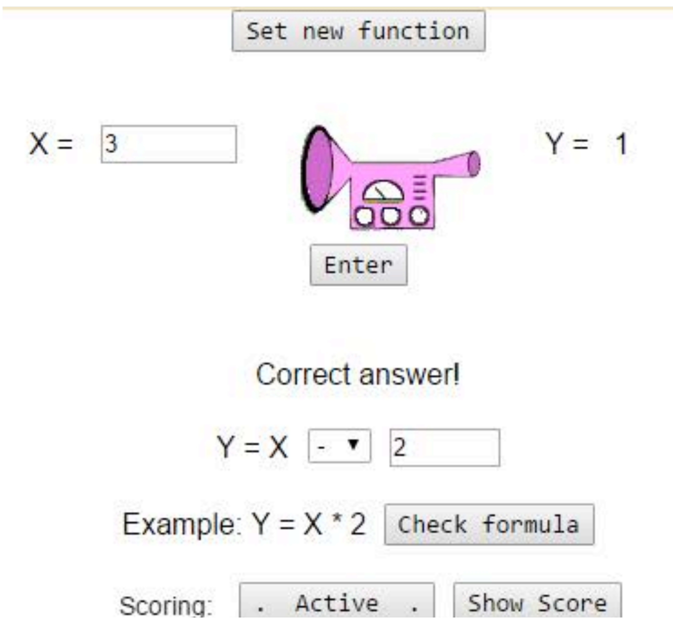

Figure 1 - *Applet* screen used

In this paper, students explore the relationships found in number patterns through arithmetic sequences using variables expressions. The task is divided into two phases and the first phase assumes exploratory contours.

In the first phase, the students fill out the X field with numbers. The *Function Machine* takes a number as *input* and give another number as *output*. The students write pairs of numbers (X and Y) in a table and deduce the algebraic expression that illustrated the pattern and/or the numerical regularity using letters to represent variables. The algebraic expression involved were the following: addition, subtraction or multiplication of a number. Later, when they had doubts, they could correct his work with the help of the *Check* button to find out whether they have answered correctly or not. If it is right, then *Correct* will appear and the students should move on to the next question. If not, the message *Try again* appears and then the answer is wrong. The click in the button *Tell me* may see the answer. This procedure was repeated six times. Ending this step, the students review the vocabulary of the Functions.

In the second phase, the task takes on contours of a problem. The students will describe the relationship found in a number pattern and express the relationship.

It was intended that students without using the *Function Machine*, write a function rule for a word problem, presented on a chart. It was pretending an algebraic expression for a function and the students must justify their solutions, using current language to describe what they think the rule is.

Data analysis was organized on two themes: to write an algebraic expression for a function with and without the *applet*, and to intend to understand the different ways of thinking of the students when they write a function rule where express the dependent variable, y, in terms of the independent variable, x.

4 RESULTS

In this presentation, we start by making a preliminary analysis on the number of correct answers. Then we analyse in more detail the writing which evidencing the mobilization capacity of generalization by students.

In the first phase, only eight students (17%) filled the tables with the pairs of numbers of the *Function Machine.* They did not make the generalization. The following graph (Figure 2) shows the number of correct answers of all the first phase items.

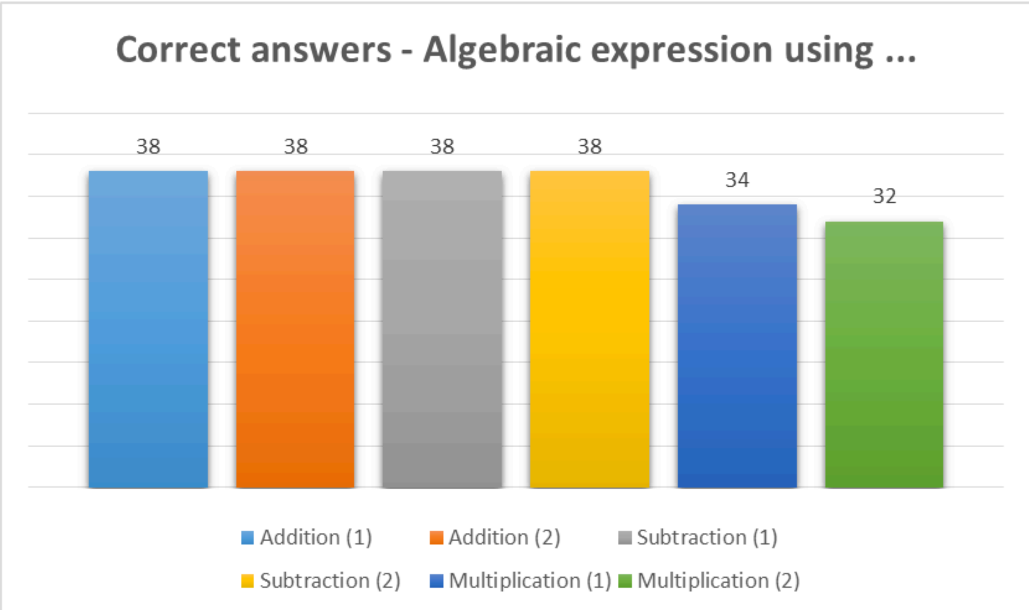

Figure 2 - Number of correct answers

A first analysis allows us to understand that the students who answered, all of them (83%) can solve tasks involving algebraic expressions with addition and subtraction. The items which registered the greatest difficulties were those in which the algebraic expressions involve multiplication (about 70%).

The graph below (Figure 3) explores the evidence of generalizability in items relating to the different numerical pattern types: addition, subtraction and multiplication. Students recognize that the pattern implies a relationship and generalize the "rule" using words and symbols or with an algebraic expression.

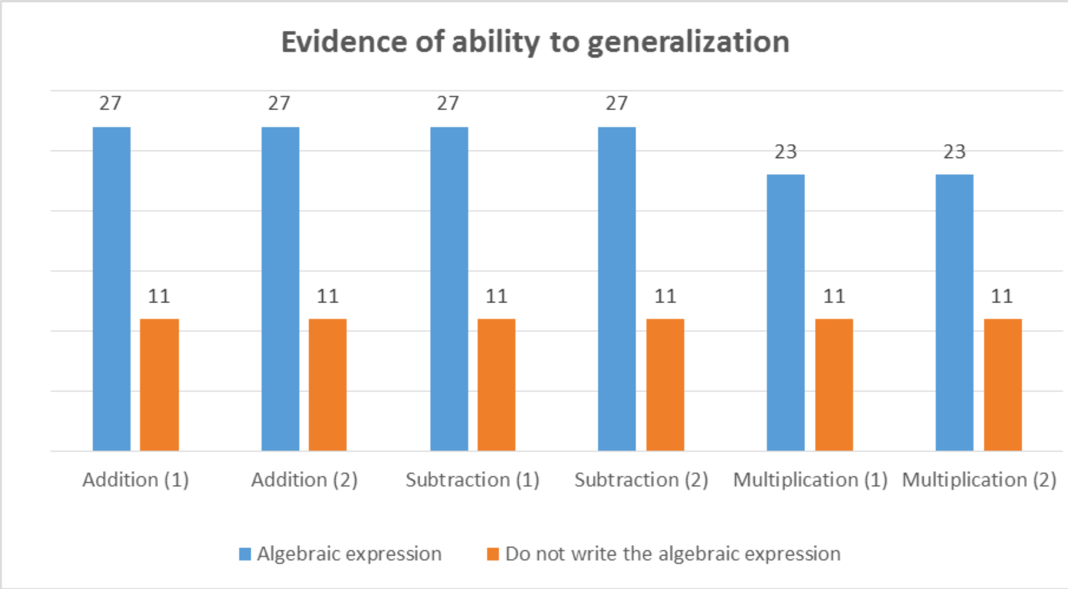

Figure 3 - Evidence of ability to generalization

Twenty-seven students (59%) wrote the algebraic expression with the addition or subtraction of a number, and twenty-three students (50%) wrote the algebraic expression with the multiplication of a number. Eleven students (24%) did not write the algebraic expression. They had difficulty to understand the meaning of variable and letters and they have been limited to write the underlying operation and the number of pattern formation rule.

In the second phase, without the use of the *applet*, twelve students (26%) did not answer the question. They did not make the generalization. The graph (Figure 4) shows the evidence of generalizability without the use of the *applet.*

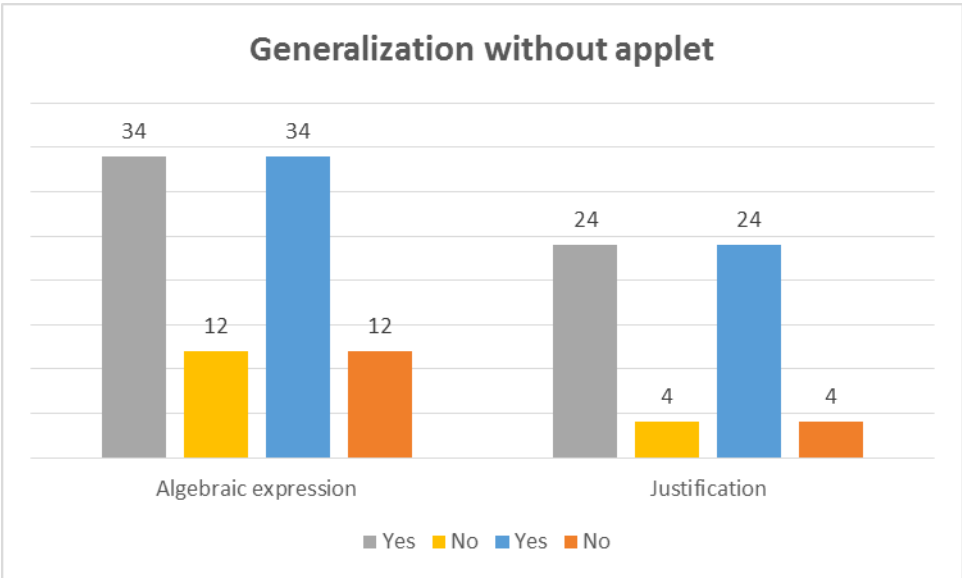

Figure 4 - Generalization without the use of the *applet*

Thirty correct answers (65%) for a total of thirty-four answers (74%) were verified. Two answers (4%) had the operation and the appropriate number to the operation, which showed the difficulty of the proper use of the variable presented by the student who knows generalize, but not write an algebraic expression.

Twenty-four students (52%) presented reasons for their answer. Twenty justifications (43%) were based on trial and error method, the students verified the values in the table. Four explanations (9%) were based on the inverse operation.

5 CONCLUSIONS

It was interesting to note that the students who participated in this exploratory study were found to have any notion of what are numerical regularities and patterns. They were able to explain the rule of formation, though the issue of the sequences and successions still had not been given on the 7th grade of basic school.

During class, students were shown to be engaged and curious to find out how to work with the *Function Machine*, having performed the tasks carefully and using the *applet* in order to confirm the answer, particularly when the numbers were large and the general expression of the pattern involved multiplication.

This study does not suggest the immediate advantages associated with the use of the *applet*, with regard to the ease of the students at the level of generalization. It was noted that the algebraic expressions involving addition or subtraction, are those where students have fewer difficulties. In some situations, students still have many difficulties in the operation of multiplication, which also revealed some weaknesses in the field of Arithmetic.

The use of activities that involve the study of patterns and regularities are one of the preferred ways to develop the algebraic thought. Standards help the students to realize the "true" variable notion, which for most is just seen as an unknown number ([25]).

This study allows us to understand the need for a longer work with the *applet*, to promote the understanding of various concepts included in the proposed tasks. On the other hand, it is necessary short-term activities with the *applet*, paper and pencil ([31]) to promote a better understanding of mathematical ideas and stimulate the justifications and generalizations. The use of *applets* can assist students in understanding mathematical concepts through visualization of its multiple representations and exploration activities ([32]).

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